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Pitch Axis Stabilization in Eccentric Orbits Using a Variable-Speed Rotor

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Introduction

A SATELLITE in a noncircular orbit stabilized by the gravitational gradient across its mass distribution undergoes planar librational motion due to the influence of the orbital motion on the attitude motion. As the eccentricity of the orbit becomes large, the amplitude of the libration increases. When the orbit eccentricity exceeds 0.355, no stable motion can exist.[1–4] This limits the use of passive gravity gradient stabilization to satellites whose orbit eccentricities are substantially less than 0.355.

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The purpose of this Note is to establish a method whereby semipassive gravity gradient stabilization can be used effectively in orbits with eccentricities significantly larger than 0.355. The method used consists of modifying the dynamical motion of the satellite by adding a small variable-speed wheel rotating about the satellite's pitch axis. The control of the wheel speed is independent of the attitude motion of the satellite, thus preserving the open-loop stabilization system and eliminating the need for attitude sensors. The speed of the wheel is dependent only on the location of the satellite in the orbit and is therefore a function only of the time since perigee passage. This technique was first considered for use in the GEOS-C satellite.[5] However, because of a decrease in the expected orbital eccentricity, it has not been implemented for that mission.

Previously, others have proposed the use of time-dependent satellite inertias for controlling satellite attitude motions.[6] However, the use of a variable-speed wheel or rotor is simpler, less expensive, and more reliable in practical applications.

Analysis

Satellite oscillations in the plane of an elliptic orbit are described by[4]

$$(1+e\cos v)(d^2\phi/dv^2)-2e\sin v(d\phi/dv)+(1/2)\delta\sin 2\phi = 2e\sin v \tag{1}$$

where e is the orbit eccentricity, $\delta = 3[(I_x-I_z)/I_y]$, ϕ is the pitch libration angle measured from the local vertical, v is the true anomaly, and I_x, I_y, I_z are the principal moments of inertia.

Consider a satellite with a variable-speed wheel rotating about the pitch axis. The moment of inertia of the rotor is defined as I_s about the spin axis, and the instantaneous speed of the wheel is Ω. The equations of motion for such a satellite in a gravitational field can be developed using the usual Lagrangian formulation. The equation for the pitch motion is

$$(1+e\cos v)(d^2\phi/dv^2)-2e\sin v(d\phi/dv)+(1/2)\delta\sin 2\phi+ I_s\dot{\Omega}/I_y\omega^2(1+e\cos v)^3 = 2e\sin v \tag{2}$$

where ω is like the mean motion and is defined as $\dot{v}/(1+e\cos v)^2$ and $(\dot{\ })$ refers to differentiation with respect to time. Equation (2) will be investigated here.

Stability Regions

The criterion chosen for stable motion is that a satellite in an elliptic orbit exhibits a motion whose maximum librational amplitude is limited to less than $\pm 90.0°$. This amplitude limitation results from the more general requirement that one axis of a satellite is defined as Earth pointing and is used for such things as communication antennas and Earth observation experiments.

The stability discussed herein is restricted to the pitch motion since the roll and yaw librations are only weakly coupled to the pitch motion and are normally stable. This is particularly true when a variable-speed wheel is used. Since the wheel speed can be controlled such that there is always a momentum bias; i.e., the wheel speed would never be zero, the roll yaw plane would be gyrostabilized provided that the pitch motion were stable. This is the familiar gyrocompass stabilization concept.

The pitch axis regions of stability can be represented in a three-dimensional space in terms of ϕ, $d\phi/dv$, and v for any given eccentricity. Since a closed-form solution to Eq. (2) has not been obtained, the stable regions must be determined numerically.[3]

The stable regions can be presented stroboscopically in two dimensions (ϕ, $d\phi/dv$) by examining the trajectories at selected values of true anomaly.[7] For example, numerically obtained values of ϕ and $d\phi/dv$ can be plotted only at each perigee crossing, apogee crossing, or at any other true anomaly v of interest. If these two-dimensional figures were to be obtained for all values of v, then they could be joined together to form a tube-shaped three-dimensional surface (see Fig. 3 of Ref. 1) or stability portrait. The portrait need only be constructed for values of true anomaly from 0 to 2π radians since, after 2π

radians, the orbit repeats itself. Finally, if the ends at $v = 0$ and $v = 2\pi$ are connected, then a doughnut-shaped surface results.

First, Eq. (1) for the pitch axis motion of a gravity gradient satellite with zero wheel speed was numerically integrated with an integration step size maintained sufficiently small to assure a maximum angular error of 0.1° after integrating over 100 orbits. A typical result is shown in Fig. 1 and confirms the work of previous researchers.[1-3]

The variable-speed rotor spinning about the pitch axis can greatly increase the regions of stability for elliptic orbits as well as increase the maximum orbital eccentricity allowed for stability. From Eq. (2) a simple control law which governs the speed of the wheel can be developed. This control is developed by observing from Eq. (2) that this equation has the form of a homogeneous differential equation if the term containing the rotor momentum is set equal to $2e \sin v$. This is the desired control law. The controlled variable is the rotor speed and the control law becomes

$$\dot{\Omega} = \frac{2e\omega^2 I_y}{I_s}(1 + e \cos v)^3 \sin v \tag{3}$$

It is noted that this control is dependent only on v and not on the pitch angle ϕ. Substituting Eq. (3) into Eq. (2) gives the governing equation for the satellite's motion

$$(1 + e \cos v)\frac{d^2\phi}{dv^2} - 2e \sin v \frac{d\phi}{dv} + \tfrac{1}{2}\delta \sin 2\phi = 0 \tag{4}$$

For the case where the eccentricity is zero, the control law vanishes as expected. Thus, the stable region of a gravity gradient satellite in a circular orbit, either with or without a controlled wheel, is identical. In Ref. 5 a similar control law was derived. However, there $\dot{\Omega}$ is a function of M, the mean anomaly, instead of v, and the control does not completely compensate for the inhomogeneous term in Eq. (2).

In real applications, the wheel speed may slowly become out of phase with v due to precession of perigee or other causes. In Ref. 8 the effect of such phase differences on the stability regions is examined.

In order to investigate the effects of the variable-speed wheel on the stability of the satellite, Eq. (4) was numerically integrated. The integration was performed for a sequence of initial conditions and was carried out over 65 orbital periods provided that at no time the pitch angle exceeded 90°. If this stability limit was exceeded, the integration was terminated and the next set of initial conditions was selected. The maximum stable trajectory then corresponds to finding the maximum initial rate or angle which will produce bounded motion when integrated over a large number of orbits.

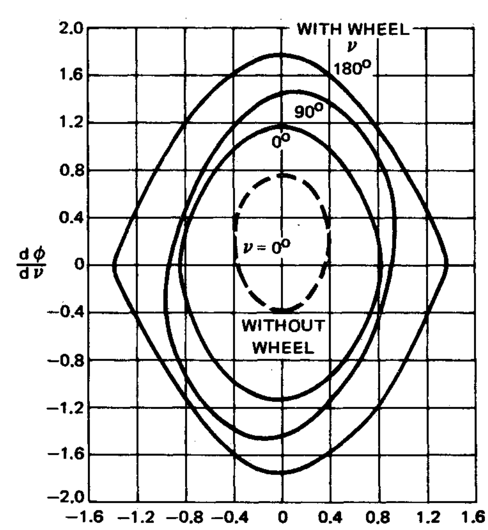

Fig. 1 Maximum stable regions for $e = 0.2$, $\delta = 3$.

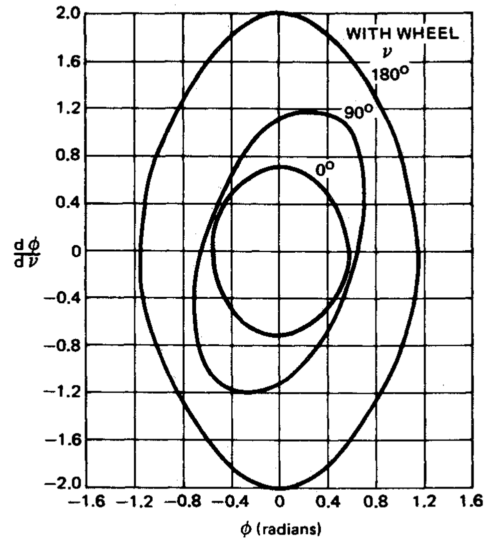

Fig. 2 Maximum stable regions for $e = 0.4$, $\delta = 3$.

These stable trajectories were evaluated by the stroboscopic method at $v = 0°$, 90°, 180°, and 270° for $\delta = 3$ and eccentricities between 0.2 and 0.8. The trajectories, when plotted stroboscopically, produce closed curves which are elliptical in shape. The major axis of the ellipse lies along the $d\phi/dv$ axis for $v = 0°$, and 180°, perigee and apogee, respectively. For $v = 90°$, the major axis of the stability ellipse is rotated clockwise, and for $v = 270°$, the major axis is rotated counterclockwise through the same angle. The plots for $v = 0°$, 90°, and 180° are shown in Fig. 1 for an eccentricity of 0.2. Also in Fig. 1, the increased size of the stability region because of the wheel can be seen for $v = 0°$.

Figures 2 and 3 show the maximum stability regions for eccentricities of 0.4 and 0.8. Stability regions for other eccentricities can be found in Ref. 9. The stability region for $v = 270°$ is not shown because it can be constructed geometrically from the stability region for $v = 90°$.

Summary

There are many distinct advantages to using a variable-speed rotor gravity gradient stabilization system. The stability regions for low eccentricity orbits are considerably larger than for satellites without the controlled rotor. This greatly aids the task of capturing the satellite within a stable region.[2] In addition, once capture is achieved, the amplitude of the librational motion

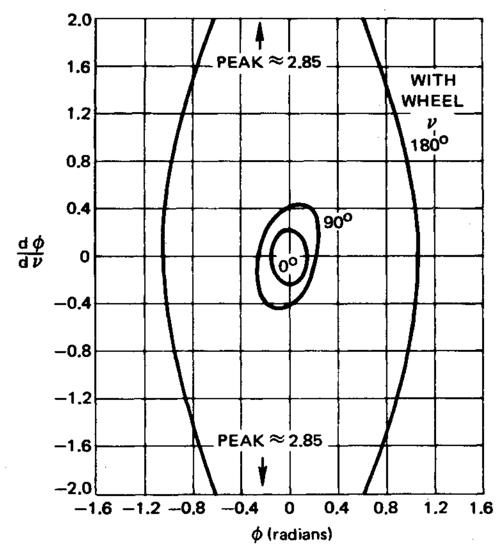

Fig. 3 Maximum stable regions for $e = 0.8$, $\delta = 3$.

is considerably smaller than if the controlled wheel were not present. For example, with an eccentricity of 0.1, a reduction of 50% in the amplitude of the natural libration can be expected.

The controlled rotor gravity gradient stabilization system offers an inexpensive open-loop approach to providing three-axis attitude control. Since attitude sensors and complex torquing devices can be eliminated, this new stabilization concept should find many applications.

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On Murphy's Stability Criterion

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Introduction

MURPHY[1] has pointed out the reason for the failure of the stability criterion of Kelley and McShane to reduce to the classical stability condition $s > 1$ (s = stability factor) while discussing the effect of overturning moment coefficient only. The same criterion is derived here afresh, using the direct method of Liapunov, and it is shown how the method reduces to that of Murphy's.

Construction of Liapunov Function

The equations of motion of a projectile[2] are written in the matrix form

$$\dot{X} = AX \tag{1}$$

where

$$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \qquad A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

x_1, x_2 are complex values of the linear and angular yaw velocity, respectively; $a_{11}, a_{12}, a_{21}, a_{22}$ are complex force and moment coefficients. Overhead dot ($\cdot$) denotes differentiation with respect to arc length.

The characteristic equation

$$|A - \lambda I| = 0 \tag{2}$$

of the matrix A has roots with negative real parts if[3]

$$\sigma_1 \leqq 0 \tag{3}$$

$$\sigma_1\sigma_2\sigma_4 + \sigma_1{}^2\sigma_3 - \sigma_4{}^2 > 0 \tag{4}$$

where σ_i's are the stability parameters, and σ_1 denotes the real part of $-(a_{11}+a_{22})$, σ_2 denotes the imaginary part of $-(a_{11}+a_{22})$, σ_3 denotes the real part of $(a_{11}a_{22}-a_{21}a_{12})$, σ_4 denotes the imaginary part of $(a_{11}a_{22}-a_{21}a_{12})$.

Let λ_1, λ_2 be the roots (complex and distinct) of Eq. (2). Now, by a theorem of Poincaré,[4] the system (1) is transformed into the canonical form

$$\begin{aligned} \dot{z}_1 &= \lambda_1 z_1 \\ \dot{z}_2 &= \lambda_2 z_2 \end{aligned} \tag{5}$$

We choose the following positive definite Hermitian form as a Liapunov function for the system (5) that is

$$L(z) = z_1\bar{z}_1 + z_2\bar{z}_2 \tag{6}$$

Its derivative

$$\dot{L}(z) = (Rl\lambda_1)z_1\bar{z}_1 + (Rl\lambda_2)z_2\bar{z}_2 \tag{7}$$

is, obviously, negative definite, subject to the conditions of Eqs. (3) and (4). Hence, the stability of the system (1) is concluded.

Murphy's Condition

Using conditions (3) and (4) and the relation

$$4(\sigma_1\sigma_2\sigma_4 + \sigma_1{}^2\sigma_3 - \sigma_4{}^2) + (\sigma_1{}^2)^2 + (\sigma_1\sigma_2 - 2\sigma_4)^2 > 0 \tag{8}$$

we derive the auxiliary condition

$$\sigma_1{}^2 + \sigma_2{}^2 + 4\sigma_3 > 0 \tag{9}$$

In the case when $\sigma_1 = 0$, the condition (9) becomes

$$\sigma_2{}^2 + 4\sigma_3 > 0 \tag{10}$$

which yields, if only the moment coefficient is considered in the notation of Murphy

$$\bar{v}^2 - 4M > 0 \tag{11}$$

that is

$$s > 1 \, (s = \bar{v}^2/4M) \tag{12}$$

Further, when spin v is zero, we have

$$M < 0 \tag{13}$$

that is

$$\phi_M < 0 \tag{14}$$

and the conclusions drawn by Murphy follow immediately.

Conclusions

As a consequence of the present investigation, it is found that, though the major stability conditions [Eqs. (3) and (4)] are essentially the same as were given by Kelley and McShane, the auxiliary condition (9), on which Murphy's discussion of the effect of overturning moment coefficient on stability would be argued, is implicit in their derivation and, hence, the reason for its failure to reduce to the gyroscopic condition $s > 1$.

Moreover, the method suggested is, also, free from the type of objection raised by Laitone.[5]

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